Pendulum Project – Report 3

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I. Introduction

In this lab report, the claims that were made by [1] about the motion of a pendulum are challenged. A pendulum is a system featuring a mass suspended by a rigid string that is proportionally negligible in weight and fixed at a pivot point on the end opposite to the mass. When a horizontal, unbalanced force is applied to the mass, such as displacing it horizontally from its point of static equilibrium, it will begin to swing with periodic motion about its point of static equilibrium. The mass will continue to swing until damping forces such as friction and air resistance in the system have decayed the maximum displacement of the pendulum to zero. Individuals wanting to study this motion thoroughly will recognize that this system is relatively complex, and so they will naturally question how this motion can be modelled mathematically. Three claims were made by [1] to describe the motion of a pendulum and motivate this lab.

First, it was predicted that pendulums can be modelled as damped harmonic oscillators which would follow Equation 1. In this equation, θ is the angular displacement, θ_0 is the angular amplitude, *t* is the time measured in seconds, τ is a variable unique to the system, T is the period, and φ_0 is a phase constant.

$$\theta(t) = \theta_0 e^{-t/\tau}\cos\left(2\pi\frac{t}{T} + \varphi_0\right) \tag{1}$$

A damped harmonic oscillator is defined as a system whose angular motion can be represented using a sinusoidal curve with exponentially decaying amplitude. At first glance, it seems reasonable to claim that a pendulum is a damped harmonic oscillator, however this claim intrinsically makes two assumptions: The period of the object's motion must be experimentally constant and independent of both the time elapsed and the initial angle of release; The decay of the pendulum's amplitude is exponential. It is proven in this lab, however, that both assumptions are only applicable at a definite range of small angles which is determined by the accuracy to which the experimenter can measure the pendulum's motion. The change in period over a set of given amplitudes was found to be best represented as a quadratic function (see Equation 2). Therefore, it was concluded that a pendulum's motion cannot be modelled as a damped harmonic oscillator.

$$T = T_0(1 + B\theta + C\theta^2) \tag{2}$$

Second, if the period is constant over all angles of release, the second prediction states that the period and length of the pendulum are related by Equation 3 where *L* is the length of the pendulum.

$$T \approx 2\sqrt{L} \tag{3}$$

This suggests that when results are fitted to Equation 4, the returned parameters should be k = 2 and n = 0.5. However, this does not reflect the results of this lab. Parameter "n" was found to be experimentally 0.5, but the k parameter was found to be equal to 2.031 ± 0.004, which is over 7 error bars above 2. This draws the conclusion that this is not a sufficient method of relating the period and length of a pendulum.

$$T = kL^n \tag{4}$$

Finally, if the period is constant over the given amplitudes and the decay of the pendulum's amplitude is exponential, the third prediction states that the quality factor (*Q*) of the pendulum can be measured using Equation 5.

$$Q = \pi\frac{\tau}{T} \tag{5}$$

For the setup used, it was found that this definition of Q factor was applicable when analyzing a given range of small angles. This was concluded by recognizing the previous results which suggested that the period is constant, and the decay is exponential within a certain range of small angles. Using this method of calculating Q factor, it was determined that the length of the pendulum has a significant effect on the system's Q factor. In fact, it was determined that this relationship was best represented by a fourth-degree polynomial (see Equation 6).

$$Q = A + BL + CL^2 + DL^3 + EL^4 \tag{6}$$

II. Experimental Setup

One pendulum system was constructed for all the experiments in this report (see Fig. 1). To form the pivot point of the pendulum, a plank of plywood was placed on top of a wardrobe in a manner that left an overhang of 20.30 ± 0.03 cm away from the edge. Then, three textbooks were placed onto the end of the plank contacting the wardrobe to ensure the system remained fixed in place. Starting 2.5 ± 0.03 cm away from the edge of the wardrobe, two holes were drilled 15.0 ± 0.03 cm apart. Three steel washers (3.1 ± 0.03 cm in diameter and 1.830 ± 0.005 mm in thickness) was selected as the mass that would be suspended by nylon fishing line. Using a kitchen scale, the mass was measured to be a total of 24.30 ± 0.05 g, and the string was measured to be 0.40 ± 0.05 g. After threading the mass through the string, both ends of the line were individually fastened in separate holes in the plywood using screws. The length of the string was determined arbitrarily with the intention to suspend the mass approximately 1 meter below the plank. Since the mass is suspended by two points, a "plumb line" made of fishing line and three more washers was used to obtain accurate measurements of the perpendicular distance between the pivot point and the mass in all stages of this lab. This distance will be referred to as the length of the pendulum in this lab. This established a system which met the requirements detailed in [1]: The pendulum used a string with proportionally negligible weight compared to the mass it suspended, it permitted adjustability in length, and its pivot point was fixed in place.

Fig. 1. Final construction of the pendulum. Angles were traced onto the white paper shown to allow experimenters to release the mass from the same reference point in each trial. Direction for positive (red arrow) and negative (blue arrow) angular displacement is shown.

In addition to meeting these base requirements, further consideration was made in experimental design to control variables that could affect the results of this lab. To begin, it was decided that the starting length of the pendulum would be constructed to be relatively long since it is generally considered that longer pendulums produce longer periods [2]. It was anticipated that constructing a pendulum with a longer period (and therefore a lower average velocity), would help reduce the effects of small changes in air resistance which may influence the data generated in this lab. Additionally, determining the change of the mass' position over certain moments in time can be identified more precisely when the mass is moving slowly. Secondly, while suspending the mass by two points instead of one made it more difficult to measure the length of the pendulum, it virtually guaranteed that the mass would not rotate, and that the pendulums motion would remain perpendicular to the observer. This also instilled more confidence when measuring angles by hand since two lengths of string had to be aligned with a protractor rather than just one, yielding a lower uncertainty in effect. Finally, the following decisions were made to measure the angular displacement of the pendulum. To measure this angle by hand, lines were traced using a protractor onto a page to be placed at the pivot point of the pendulum (see Fig. 1). To measure angles using [3], a software which can track the motion of an object in a video, the mass was coloured black, and a white background was setup behind the path of motion of the mass as shown in Fig. 2. Importance was also placed on ensuring that the videos used with this software were recorded at a constant frame rate since the software does not account for a variable frame rate when calculating the elapsed time that corresponds to each measured position of the mass.

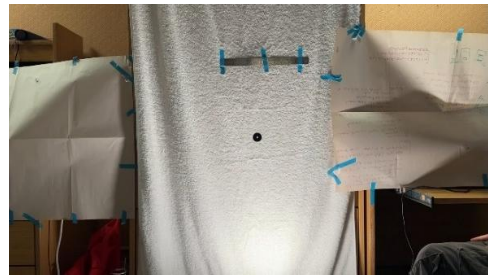

Fig. 2. A white background was added behind the pendulum to facilitate auto-tracking. This image also shows the direction in which Q factor was measured.

III. Methods

First, it was measured that the length of the pendulum was 0.9620 ± 0.0005 m. Using this length, the following experiments were performed to determine the relationship between angle and period, and then the relationship between angular amplitude and elapsed time.

The pendulum was raised to 1.40 ± 0.03 rad from its position at equilibrium and then released. Ten oscillations were timed to render the uncertainty posed by the experimenter's reaction time proportionally negligible to the period measured. This was repeated for three separate trials to obtain an average measured period for this specific angle. This process was repeated for angles ranging 1.22 ± 0.03 rad to 0.35 ± 0.03 rad by increments of 0.02 ± 0.03 rad, and then repeated on the other side of the pendulum to test negative angles for asymmetry. Data was recorded in Table I of Appendix A.

Beginning at the largest angular displacement that fit within the bounds of the white background, the pendulum was recorded at 59.96 FPS until the amplitude of the pendulum was very small and the decrease in amplitude after every oscillation was unnoticeable. This video was imported into the program "Tracker" [3]. The grayscale and brightness filters were applied to help the tracking software better differentiate between the mass and the background. Scale was established using a 30 cm ruler that had been taped to the background so that it would be in the frame of the video. The program used this video to measure, in meters, the x-position of the mass in every frame. Using this data, angular displacement was calculated. First, the x-positions relative to the position of equilibrium were calculated by subtracting the mean x-position from each individual x-position. Then, the arcsine function was applied to the ratio between each "true x-position" and the originally measured pendulum length to represent these positions as angular displacement. Finally, the local extrema of the pendulum's angular displacement were determined from these results. The absolute value of these local extrema provided values for the amplitude of the pendulum's swing which was recorded in Table II of Appendix A along with the corresponding values for time elapsed.

It was found from the results of the experiments previously described that the relationship between period and angle is experimentally constant between a certain range of small angles. The range associated with the uncertainties of the auto-tracking method was found to be between -0.28 ± 0.02 rad and 0.28 ± 0.02 rad (To see how this was found, see Appendix C).

To find both the relationship between the pendulum length and the period, and the relationship between the pendulum length and the q factor, the following experiment was performed. The length of the pendulum was first adjusted to be 1.1000 ± 0.0005 m by removing one screw, puling the fishing line to the desired height, and then refastening the screw. Then, the amplitude decay was measured using "Tracker" in the same way that previously described. This was repeated seven more times where the length was decreased between each trial. It was aimed for the eighth trial to have a length of approximately 15% of the starting length (actual measurement of 0.1630 ± 0.0005 m). The change in length between each trial was generally constant, however, it was aimed to have smaller increments for the first three repetitions where the ratio between the size of the mass and the length of the pendulum would be smaller. The data generated at these longer lengths would therefore have lower uncertainties.

Starting at amplitudes within the small angle range, multiple measurements of period were obtained for each length as this is represented as the time elapsed between each neighbouring pair of local maxima, and each neighbouring pair of local minima. This was recorded in Tables IV-XI of Appendix A.

Upon inspection of the results from measuring Q factor using both methods as instructed by [1], it was determined that measuring Q factor using Equation 5 was more accurate, as it calls for amplitudes that decay exponentially. From the results of previously described experiments, the decay can be represented by an exponential function at smaller angles, and so this method is suitable for this setup.

IV. Results

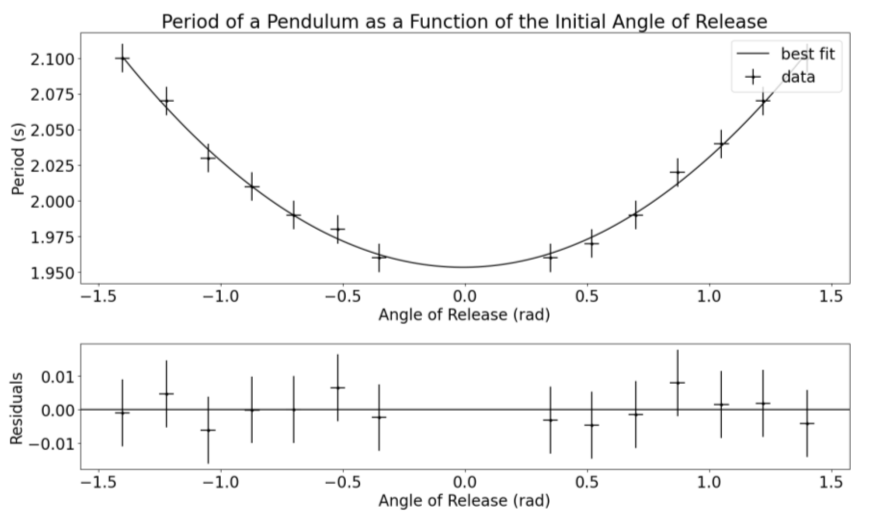

Fig. 3. Identifies the relationship between angles of release and the average recorded period for a pendulum. Release angles ranging from -1.396 rad to +1.396 rad were measured by hand, producing an uncertainty of ± 0.03 rad. Period was measured as an average using a handheld timer as described in *I. Procedure*, creating an uncertainty of ± 0.01 seconds. The data was fit to a quadratic curve using [4] and [5] (Using Equation 2: $C = 0.038 \pm 0.002$, $B = 0.000 \pm 0.001$, $T_0 = 1.953 \pm 0.005$) which sufficiently describes the trend. As a result, this data disagrees with the predictions from Equation 1 since this pendulum consistently demonstrates a non-constant period over different angles.

A. Angle and Period Measurements Performed by Hand.

When determining the uncertainties for angle, there were only measurable Type B uncertainties. This was determined by tracing angle markers onto a page using a protractor which measured in increments of 1 degree, and then aligning the pendulum to the desired angle re releasing the pendulum. As a result, it was determined that angle could be measured with an uncertainty of ± 1 degrees which was divided by 2 in accordance with course expectations. This produced the uncertainty of ± 0.03 rad.

When determining the uncertainties for period, there were Type B and Type A uncertainties. The Type B uncertainty can be represented by the reaction time of the experimenter which was measured to be an average of approximately 0.1 seconds (see Table III in Appendix A). The Type A uncertainty was calculated to be approximately 0.003 seconds by calculating the ratio between the standard deviation of the measured time and the square root of the number of measurements. In accordance with course expectations, the highest uncertainty of 0.1 seconds was accepted. This uncertainty was further reduced by a factor of 10 since 10 oscillations were measured in each trial when this method was used.

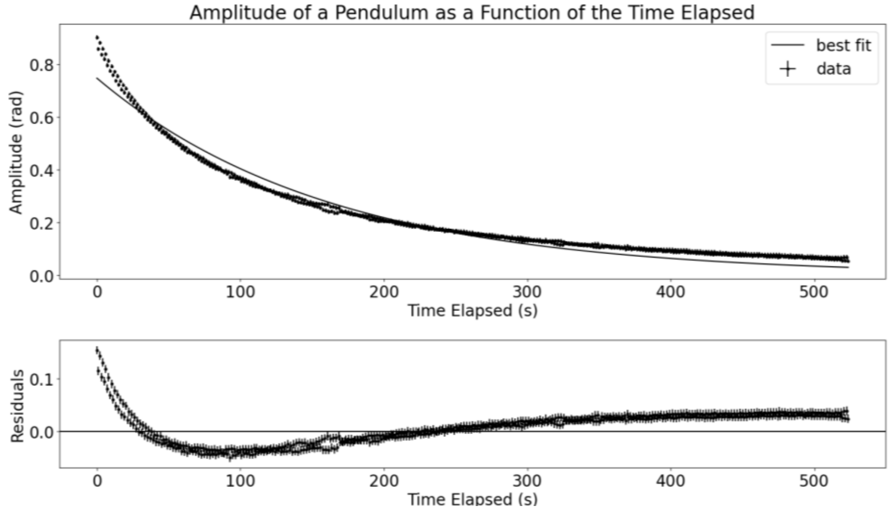

Fig. 4. Depicts the amplitude of a pendulum's oscillation starting at an angle of 0.85 ± 0.02 rads over a particular time interval (see Table II in Appendix A for raw data). Comparing this fit to the exponential component of Equation 1 using [4] and [5], t = 0.746 ± 0.001 and τ = 162.5 ± 0.4. Since this was done using a frame-by-frame analysis of a 59.96-FPS video recording of the pendulum's motion, the horizontal error bars are not visible as they are representing the ± 0.008 seconds of uncertainty in the measurement of time. In contrast, the vertical error bars represent the uncertainty of in the pendulum's amplitude which is represented differently for each point. As shown by the residuals, fitting this data to an exponential function does not sufficiently describe the pattern formed by the decay of the pendulum's amplitude.

B. Angle and Period Measurements Performed Using Tracker

When determining the uncertainty in elapsed time, there were only measurable Type B uncertainties. In accordance with course expectations, this is represented as the lowest interval of measurement of time divided by 2. For the 59.96 Hz video recorded, the uncertainty was therefore approximately ± 0.008 seconds.

To determine the uncertainty in measured angle there were only measurable Type B uncertainties. To determine this uncertainty, it was first required that the uncertainties of the x-position and the length of the pendulum were determined. The uncertainty of the x-position of the mass could be calculated by considering that the smallest interval of measurement of these values was the diameter of the mass itself to account for the "motion distortion" in each frame as shown in Fig. 5. The uncertainty of the length was determined as the smallest interval of measurement (0.001 m) divided by 2. To calculate the angle of the pendulum, the x-position of the mass must be divided by the length. As a result, it is necessary to determine the largest percentage uncertainty between these values and multiply this ratio by this percentage accordingly. Then, the arcsine function was applied to this ratio and its uncertainty to receive each angle and its respective uncertainty.

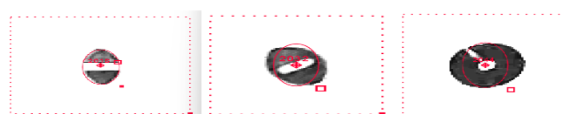

Fig. 5. Each frame of the video recorded of the pendulum had some distortion which increased the uncertainty in the mass' x-position.

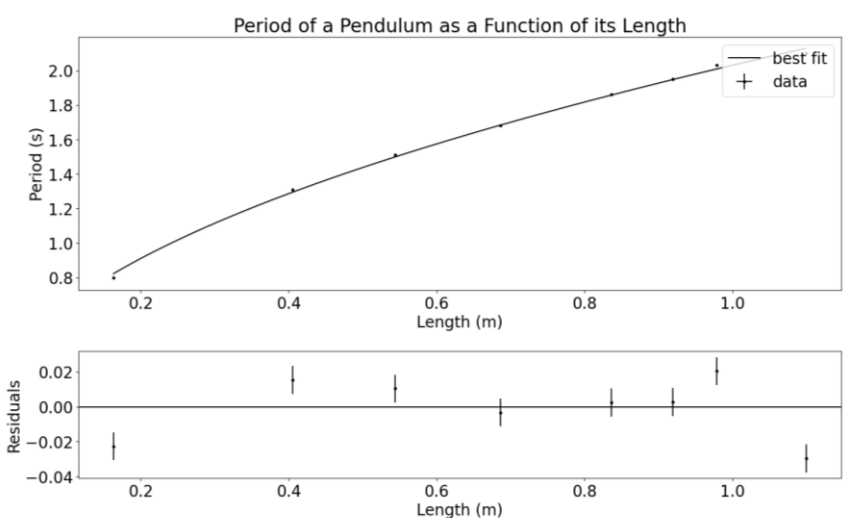

Fig. 6. Plots the relationship between the period and the length of the pendulum for lengths between 0.1630 ± 0.0005 m and 1.1000 ± 0.0005 m. Using [4] and [5], the data used for this graph was fitted to Equation 4 where k = 2.031 ± 0.004 and n = 0.498 ± 0.004. On this plot, the vertical error bars are small, but they represent an uncertainty of ± 0.008 seconds resulting from the frame rate of 59.96 FPS that was used when recording the videos. The horizontal error bars are also too small to be visible, but they represent an uncertainty of ± 0.0005 m to reflect the uncertainty of measuring the length with a ruler that measured with an accuracy of 1 mm.

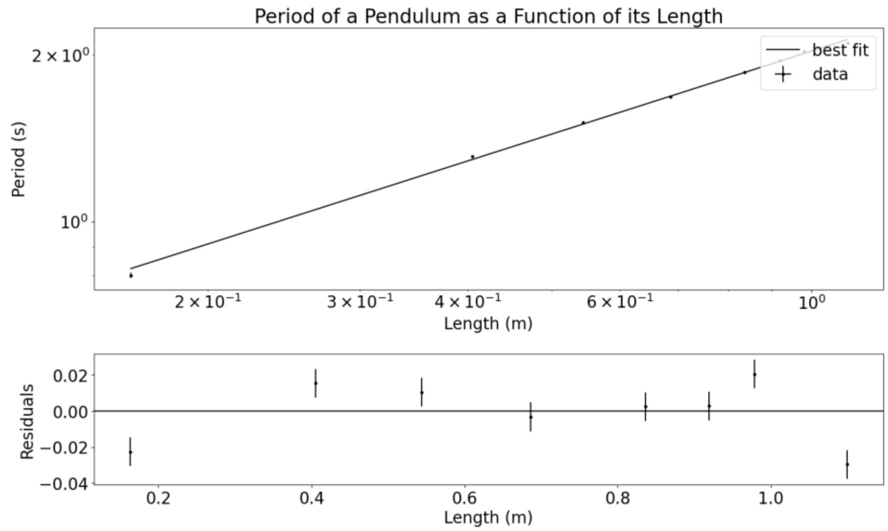

Fig. 7. Plots the relationship between the period and the length of the pendulum on a log-log plot for lengths between 0.1630 ± 0.0005 m and 1.1000 ± 0.0005 m. Using [4] and [5], the data used for this graph was fitted to Equation 4 where k = 2.031 ± 0.004 and n = 0.498 ± 0.004. On this plot, the vertical error bars are small, but they represent an uncertainty of ± 0.008 seconds resulting from the frame rate of 59.96 FPS that was used when recording the videos. The horizontal error bars are also too small to be visible, but they represent an uncertainty of ± 0.0005 m to reflect the uncertainty of measuring the length with a ruler that measured with an accuracy of 1 mm.

C. Period Uncertainties used for Fig. 7.

Since it was considered that the period is constant at the angles measured for this experiment, it is important to note that type A uncertainties can also be calculated for the period measurements. However, for all periods measured, the type b uncertainty was larger.

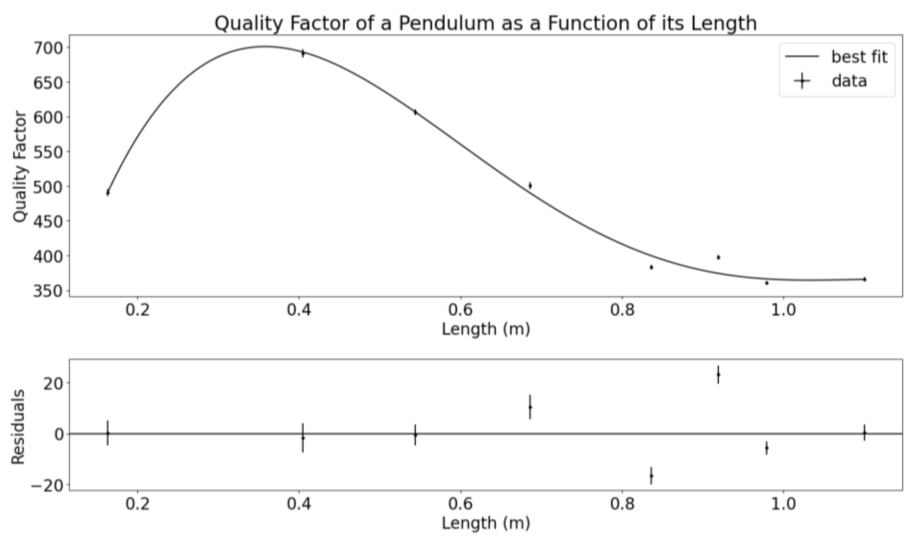

Fig. 8. Shows the relationship between the measured Q factor and the length of the pendulum for lengths that range from 0.1630 ± 0.0005 m to 1.1000 ± 0.0005 m. This graph was plotted using [4] and [5]. Upon comparing 5 different fitting functions (see Appendix D), it was determined that the trend was best represented by fitting the data to a fourth-degree polynomial such as the one described by Equation 6 where A = -210 ± 30, B = 6300 ± 300, C = -14800 ± 900, D = 13000 ± 1000, and E = -3800 ± 400. On this plot, the small, vertical error bars represent the uncertainty calculated for the measurement of Q factor which varies for each point (ranges from ± 3 to ± 6, see Table XIII of Appendix A). In contrast, the horizontal error bars are constant for all points, however, they are too small to be visible. The horizontal error bars represent an uncertainty of ± 0.0005 m to reflect the uncertainty of measuring length with a ruler to the nearest mm. These results indicate that the Q factor of the pendulum is strongly dependent on the length of the pendulum and as a result, there is a length between 1.1000 ± 0.0005 m and 0.9790 ± 0.0005 m where Q factor is maximized for this setup.

D. Q factor Uncertainties Used for Fig. 8.

Since Q factor was measured using Equation 5, the uncertainty was determined by comparing the percentage uncertainties of the period and the calculated tau value at each point. In accordance with course expectations, the largest percentage uncertainty was taken and applied to the calculated Q factor. This resulted in a Q factor unique to each point.

V. Analysis and Discussion

A. Period vs Angle

As shown in Fig. 3, the trend in the data measured is best represented when fit with a quadratic function (see Equation 2). This was done using [4] and [5] which outputted values for *C*, *B*, and T_0. The value of B was found to be experimentally zero since it was calculated to be less than its uncertainty. This is an important result because it confirms that any asymmetrical characteristics of the pendulum's motion were negligible.

Evidently, this result suggests that there is a relationship between the angle of release and the period of the pendulum's swing. However, this data suggests that the period becomes experimentally constant at smaller angles. Data points below 0.35 rad would not have reinforced this conclusion since the error bars span below the vertex of the line of best fit. Following this conclusion, it was found that when using tracker, angles within -0.28 ± 0.02 rad to 0.28 ± 0.02 rad will yield periods that are experimentally constant (see Appendix C)

B. Angular Amplitude vs Time Elapsed

Fig. 4 provides a clear indication that the pendulum's amplitude decays as time progresses since its initial release. The rate at which this decay occurs is still unknown since the data did not fit an exponential curve.

Q factor must be determined using both methods, as instructed by [1]:

1. According to [1], Q/n can be determined as the number of oscillations it takes for the amplitude of the pendulum's motion to decay to ($e^{-\pi/n}$ *100) % of the original amplitude. From the results of *B.,* the starting amplitude was 0.901 ± 0.008 rad and the final amplitude was 0.054 ± 0.008 rad. Therefore, Q factor is calculated to be 593 ± 2 (see Appendix B).

2. The second method requires the use of Equation 5 [1]. Based on the mathematical model shown in Equation 1, it was determined that τ is equal to 162.5 ± 0.4 seconds using a fitted exponential function to the data points in Table II (see Appendix A). Similarly, period was calculated to be 1.968 ± 0.008 seconds by taking the average period of these data points. Using these values, Q factor was calculated to be 259 ± 1.

These Q factors vary greatly from one another (a difference of 167 error bars). This is likely since both methods assume that the decay of the pendulum's amplitude is exponential. Additionally, method #2 also makes use of a period that is assumed to be constant over the entire pendulum suggesting the conclusion that this method is less accurate. However, since neither method requires a specific range of amplitudes, a range of small angles were used in the experiments that followed that render both assumptions to be experimentally true. Now that both methods are applicable, method #2 is chosen to calculate Q factor since it conforms to the entire sample size whereas method #1 only considers the first and last recorded data points.

C. Period vs Length

As shown in Fig. 7., a power law function has some correlation to the relationship between the period and the length of the pendulum. As indicated by the residuals however, this fit is not perfect as some points lie a significant amount of error bars away from the line of best fit. Moreover, while the parameter "n" in Equation 4 was experimentally equal to 0.5, the parameter "k" was not experimentally equal to 2 as predicted by [1]. In fact, the value for "k" was over 7 error bars above this predicted value. Upon further analysis, Equation 3 does not equate when imputing the associated units of each variable, suggesting there is likely another variable or a universal constant that was deemed negligible. As demonstrated by these results, this was a poor assumption.

D. Q Factor vs Length

In Fig. 8., it is shown that the change in the calculated Q factor between adjacent points is generally on a scale of over 10 error bars. This undeniably suggests that the measured Q factor of the pendulum is dependent on the length of a pendulum. Furthermore, after testing multiple best fit functions (see Appendix D), it was found that this relationship was best represented by a fourth-degree polynomial (see Equation 6). This was concluded since the other fit functions that were tested outputted higher uncertainties, and featured residuals where data points still formed a pattern. These are signs that the function was not representing the full relationship between the variables that were tested. The fitting of this curve results in a local maximum between the lengths of 1.1000 ± 0.0005 m and 0.9790 ± 0.0005 m. This indicates that the Q factor can be maximized for this given setup if the length is changed to the optimal length that would be found within this range. It is worth noting however, that this fit was made using 5 parameters to 8 data points. This calls the credibility of these results into question. To achieve more credible results, one must repeat this experiment for more pendulum lengths.

VI. Uncertainties

In this lab, some assumptions were made about the experimental setup and methods of testing that could have affected certain outcomes of the experiments.

One potential source of uncertainty in the results presented is found in the securement of the string ends at the pivot of the pendulum. As explained in section II, the pivot was established by screwing the string ends into a wooden board. By choosing to use this setup for these experiments, the assumption was implicitly made that the string would not be rubbing against the board or screw (generating friction), nor would it be slipping out of the hole (causing an increase in length). An attempt was made to avoid these effects by using new and reliable materials, however, a more secure system could have been constructed by sacrificing the feasibility at which the length could be manually adjusted. Additionally, unpredictable variations in the testing environment could have been identified and addressed by performing more trials which would permit the calculation of Type A uncertainties.

An increase in unpredictable friction within the system would have an impact on the decay of the pendulum's amplitude which was shown in Fig. 4. Similarly, any significant increase in length during the testing would impact the numerical results from the experiments summarized by Fig. 3, and Fig. 5-7. The experiments associated with these figures tested the period, and quality factor of the pendulum, both of which were concluded to have some relationship with the length of the pendulum.

VII. Conclusion

In conclusion, the results of this lab clearly demonstrate that the claims made by [1] feature mistaken assumptions about the motion of a pendulum.

First, contrary to what was assumed, the period of a pendulum is not independent of its amplitude and its amplitude does not decay exponentially for angles between $\frac{\pi}{2}$ and 0 rad. Accordingly, this disagrees with the provided mathematical model that is described in Equation 1. However, it was found that both assumptions become valid at a smaller range of angles that is determined by the accuracy to which the experimenter can measure the pendulum's motion. Using the tracker method, this range of angles was calculated to be between -0.28 ± 0.02 rad and 0.28 ± 0.02 rad.

As a consequence of these incorrect assumptions, the two suggested methods of measuring Q factor disagree as they both rely on the mathematical model correctly predicting the motion of the pendulum. To obtain the most accurate measurement of Q factor for this pendulum, it was determined that method #2 would be used since at the small range of amplitudes, both assumptions become experimentally true. With both methods deemed applicable, method #2 was chosen to calculate Q factor because it conforms to the entire sample size whereas method #1 only considers the first and last recorded data points.

Even after using methods that addressed the previous mistaken assumptions, it was found that the period and length of the pendulum cannot be related by Equation 3, as predicted by [1].

Finally, it was found that the Q factor of the pendulum was dependent on the length. Furthermore, the found relationship suggested that there is a length between 1.1000 ± 0.0005 m and 0.9790 ± 0.0005 m which would maximize the Q factor of the pendulum.

To achieve more specific and definitive conclusions, more data must be taken, primarily if more definitive conclusions are desired regarding the relationship between the Q factor and length of the pendulum. However, as previously discussed, data measured for smaller lengths will feature greater uncertainties. Therefore, to proceed, a smaller mass must be used, and future videos must be recorded at higher frame rates to obtain data that is still conclusive.

Appendix A

Table I. Raw data from *Period vs Angle*

Table II. Raw data from *Angular Amplitude vs Time Elapsed*

Table III. Reaction time of experimenter was determined by taking the average of 20 trials. In each trial, the goal of the experimenter was to start the timer used in *Part 1: Period vs Angle* and then stop it as close to 2.00 seconds as possible. Each time was recorded. The difference was considered the reaction time, and a mean reaction time was calculated to quantify the Type B uncertainty for the periods measured in *Part 1: Period vs Angle*.

Table IV - XI. Raw data from pendulum videos recorded for different lengths.

Table XII. Data calculated for *Period vs Length*

Table XIII. Data calculated for *Q Factor vs Length*

Appendix B

Calculating Q factor using method #1.

$$0.901 \times e^{-\frac{\pi}{n}} = 0.054$$

$$\ln(0.901 \times e^{-\frac{\pi}{n}}) = \ln(0.054)$$

$$\ln(0.901) + \left(-\frac{\pi}{n} \times \ln(\mathrm{e})\right) = \ln(0.054)$$

$$\ln(0.901) - \frac{\pi}{n} \times \ln(\mathrm{e}) = \ln(0.054)$$

$$n = -\frac{\pi}{\ln(0.054) - \ln(0.901)}$$

$$\#\text{ of oscillations} = \frac{Q}{n}$$

$$\#\text{ of oscillations} \times n = Q$$

$$532 \times \left(-\frac{\pi}{\ln(0.054) - \ln(0.901)}\right) = Q$$

$$\therefore \mathrm{Q} = 593$$

Calculating uncertainty for the Q factor measured using method #1 [7], [8].

$$\sigma_n = \pm\left|\frac{\pi}{\max\left[\left(\pm\frac{0.008}{0.054}\right), \left(\pm\frac{0.008}{0.901}\right)\right]}\right|$$

$$\sigma_n = \pm\left|\frac{\pi}{\max[(\pm 10\%), (\pm 0.9\%)]}\right|$$

$$\sigma_n = \pm 0.3\%$$

57 positions were measured during this last oscillation

$$\rightarrow \sigma_{\#\ of\ oscillations} = \pm\frac{1}{57} \div 2$$

$$\rightarrow \sigma_{\#\ of\ oscillations} = \pm 0.009$$

$$\therefore \sigma_Q = Q \times \max[\pm 0.3\%, \pm 0.001\%]$$

$$\therefore \sigma_Q = \pm 2$$

Appendix C

Calculating the range of angles where the period is experimentally constant according to uncertainties generated in auto-tracking method:

Obtain auto-tracked data representing the period vs amplitude relationship from Table II, Appendix A. Then fit to Equation 2 using [4] and [5]. This generated the following fit parameters:

T_0 = 1.9597208694222716 +/- 0.00002037275154572386 4

B = 0.0007285943931826244 +/- 0.003056680125548378

C = 0.05216760171605739 +/- 0.0006491686283426898

The local minimum of this function can therefore be calculated to be ~ (0.00698 rad, 1.959734559 s). Then the uncertainty of measuring time with "Tracker" [3] (0.008 s) was added which indicated that angles on the quadratic fit function, whose corresponding period lies below the line of T = 1.967734559 would generate experimentally constant periods. Intersecting this line with the quadratic function found that angles above -0.313683 rad, and below ~0.300273 rad generate periods that are experimentally constant. To account for the uncertainty in the ability to measure angles using this method (highest was 0.02 rad), it was considered that for the pendulum lengths measured, angles within -0.28 ± 0.02 to 0.28 ± 0.02 will yield periods that are experimentally constant.

Appendix D

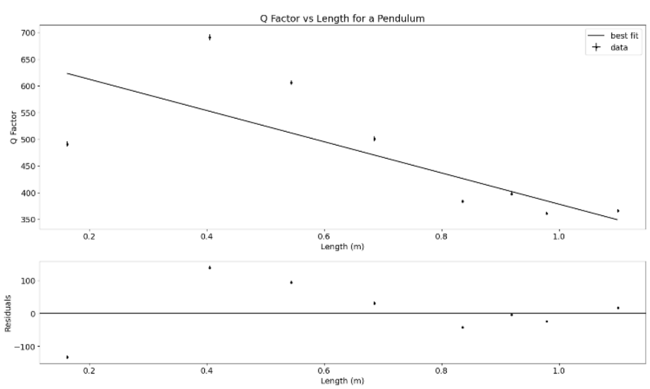

Fig. 9. Same data as Fig. 7. but it was fitted to a linear function instead.

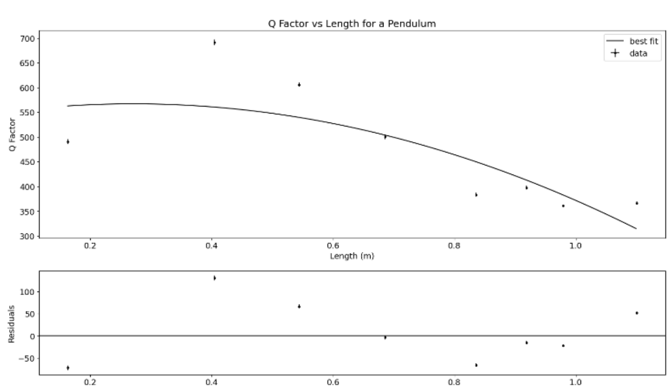

Fig. 10. Same data as Fig. 7. but it was fitted to a quadratic function instead.

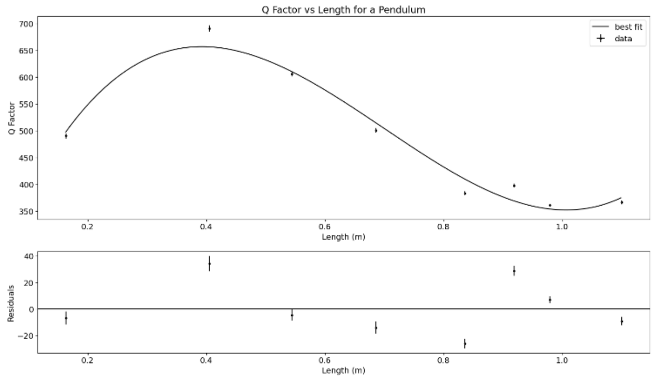

Fig. 11. Same data as Fig. 7. but it was fitted to a third-degree polynomial function instead.

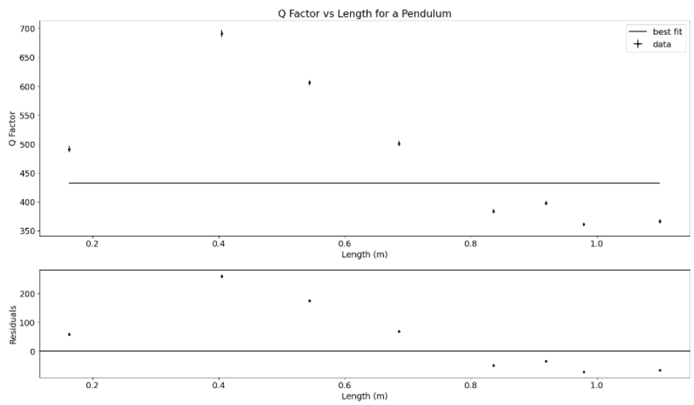

Fig. 12. Same data as Fig. 7. but it was fitted to a exponential function instead.

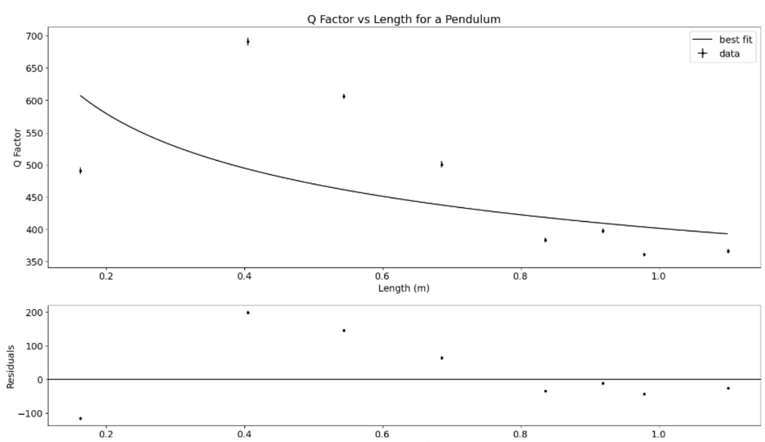

Fig. 13. Same data as Fig. 7. but it was fitted to a power law function instead.

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